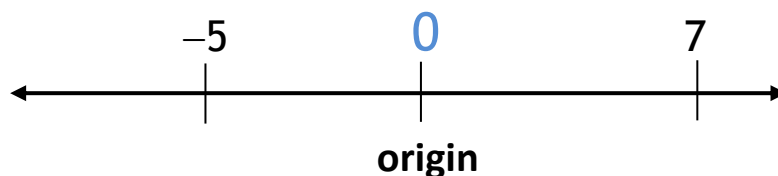

EQUATIONS – DISTANCE AND THE ORIGIN

Sometimes the “size” of a number is more important than whether it’s positive, zero, or negative. To measure the size of a number, we consider the **distance** between the number and zero (the origin) on the number line.

□ EXAMPLES



What is the distance from the number 7 to 0? It’s 7.

How far from the origin is the number -5? It’s 5 units away.

What is the distance from the origin to 2π ? The distance is 2π .

What is the distance between the number 0 and the origin? It’s 0.

□ NOTES

1. We can calculate the distance from any number to 0 (the origin). In other words, the distance between the origin and any point on the line always exists, even if it’s hard or impossible to calculate.
2. Distance must be greater than or equal to 0; that is, it’s never negative. If d represents distance, then $d \geq 0$.

Homework

1. For each number, calculate its distance to 0 (the origin):
a. 18 b. -23 c. 0 d. $\frac{2}{3}$ e. $-2\frac{3}{5}$
f. π g. $-\pi$ h. $\sqrt{5}$ i. $-\sqrt{17}$ f. $\sqrt{-9}$
2. For each expression, calculate its distance to 0 (the origin):
a. $20 - 12$ b. $1 - 9$ c. $3.7 - 3.7$ d. $0 - 17$
e. $\pi - \pi$ f. $\sqrt{2} + \sqrt{2}$ g. $\pi + \sqrt{2}$ h. $0.5 - 2$
3. If d represents the distance from any point on the line to the origin, then
a. $d > 0$ b. $d \geq 0$ c. $d < 0$ d. $d \leq 0$
4. T/F: Given any number, we can always calculate its distance to the origin.
5. T/F: Given any number, its distance to the origin is always positive.
6. T/F: Given any number, its distance to the origin is never negative.
7. T/F: The distance between 0 and the origin does not exist.

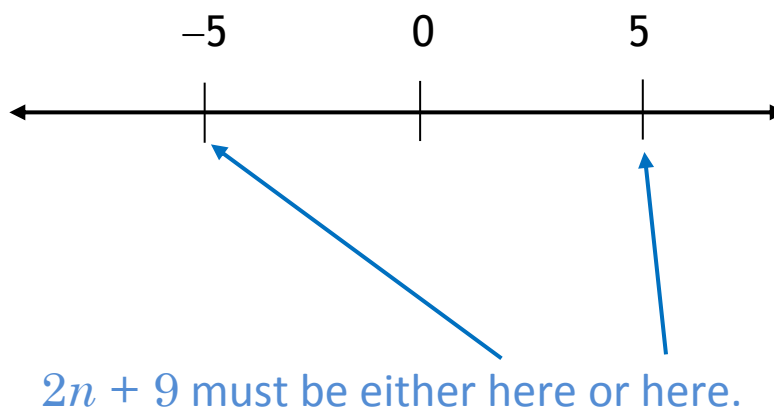
□ **EXAMPLE**

I'm thinking of a number (or numbers). If the number is doubled and then 9 is added, the final result will be 5 units from the origin.

Solution: How about we let n represent the unknown number (or numbers). The problem states that

"the number is doubled and then 9 is added"

This sequence of events transforms the n into the expression $2n + 9$, and it is this quantity which must be 5 units from the origin. Well, what does it really mean for a quantity to be 5 units from the origin? It means that the quantity must be either $+5$ or -5 (since both $+5$ and -5 are 5 units from the origin).



Writing this idea with equations, we get two possibilities:

$$2n + 9 = -5$$

$$2n = -14$$

$$n = -7$$

$$2n + 9 = 5$$

$$2n = -4$$

$$n = -2$$

Therefore, the numbers I was thinking of were -7 and -2

Homework

8.
 - a. I'm 7 units from the origin on the number line. What number(s) am I?
 - b. The distance between me and 0 on the number line is 100. What am I?
 - c. I'm 0 units from the origin. What number(s) can I be?
 - d. The distance between me and the number 0 is 0. Who am I?
 - e. The distance between me and the origin is -10 . What number(s) can I be?
 - f. I'm -4 units from 0 on the number line. Who am I?
9. I'm thinking of a number or two...
 - a. If 1 is added to the number, you get a number whose distance to the origin is 9.
 - b. If 3 is subtracted from the number, the result is 10 units from 0 on the number line.
 - c. If the number is doubled, the resulting number is 12 units from the origin.
 - d. If the number is tripled, you get a number whose distance to the origin is 15.
 - e. If 7 is subtracted from the number, the resulting number is 0 units away from 0.
 - f. If 5 is added to the number, the distance between the final result and the origin is 0.
10. I'm thinking of a number or two...

- a. If 3 is added to the number, you get a number whose distance to the origin is 14.
 - b. If 5 is subtracted from the number, the result is 12 units from 0 on the number line.
 - c. If the number is doubled, the resulting number is 8 units from the origin.
 - d. If the number is tripled, you get a number whose distance to the origin is 9.
 - e. If 4 is subtracted from the number, the resulting number is 0 units from the origin.
-
- 11. I'm thinking of a number (or numbers). If the number is quadrupled and then 8 is subtracted, the final result will be 24 units from the origin.
 - 12. I'm thinking of a number (or numbers). If the number is tripled and then 6 is added, the final result will be 9 units from the origin.
 - 13. I'm thinking of a number (or numbers). If the number is doubled and then 9 is subtracted, the final result will be 10 units from the origin.
 - 14. I'm thinking of a number (or numbers). If the number is quintupled and then π is added, the final result will be -7 units from the origin.

Solutions

1. a. 18 b. 23 c. 0 d. $\frac{2}{3}$ e. $2\frac{3}{5}$

f. π g. π h. $\sqrt{5}$ i. $\sqrt{17}$ f. Not a number

2. a. 8 b. 8 c. 0 d. 17

e. 0 f. $2\sqrt{2}$ g. $\pi + \sqrt{2}$ h. 1.5

3. b. **4.** T **5.** F **6.** T **7.** F

8. a. 7 and -7 b. 100 and -100 c. 0 d. 0
 e. Impossible; no distance can be -10 .
 f. Impossible; no distance can be -4 .

9. a. -10 and 8 b. -7 and 13 c. 6 and -6
 d. 5 and -5 e. 7 f. -5

10. a. 11 and -17 b. 17 and -7 c. 4 and -4
 d. 3 and -3 e. 4

11. a. 8 and -4 **12.** 1 and -5

13. $\frac{19}{2}$ and $-\frac{1}{2}$ **14.** Impossible

“Next in importance to freedom and justice is education, without which neither freedom nor justice can be permanently maintained.”

President James A. Garfield (1831 - 1881)